Search, Liquidity, and Retention:

Screening Multidimensional Private Information*

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Abstract

A large literature has shown that asset quality may be signaled by retention, or, in a separate literature, screened by liquidity. I present a general framework to analyze both instruments, offering conditions that characterize which instrument will be used in equilibrium. I then expand the private information to include not only asset quality but also seller patience, showing that both retention and liquidity may be used to fully separate both dimensions of private information. The expanded model offers new predictions about how price, quantity, and liquidity covary with each other and with seller private information.

JEL codes: D45, D82, D83, D86, G12, G23, G32

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1 Introduction

Ever since Leland and Pyle (1977), a large body of finance literature has studied retention signaling, the idea that when only part of a divisible asset is sold, the unsold quantity can signal the asset's privately known quality. For example, a firm divesting a profitable subsidiary receives a high payoff from its unsold holdings, so it is more willing to retain a high fraction of ownership in the unit. As a result, selling a low quantity functions as a credible signal of quality.

A more recent literature has used competitive search to illustrate another way that buyers may identify asset quality: sellers with good assets may charge a high price, inducing a low *probability* of trade, which this literature calls low *liquidity*. The intuition for the liquidity channel is similar to retention signaling: sellers of good assets are willing to suffer a low probability of sale, because if they fail to find a buyer they still receive a high payoff from the asset.

Although the intuition underlying retention and liquidity is similar, the relationship between them is not well understood. The contribution of this paper is to present a general framework that jointly analyzes both instruments. I begin by embedding a standard retention model with private asset quality in a competitive search environment, and present sufficient conditions that characterize which screening instrument will be used in equilibrium. I then expand the space of private information to include not only asset quality but also seller patience, showing that there exists an equilibrium in which retention and liquidity work together to fully separate both dimensions of private information. The expanded model offers new predictions about how price, quantity, and liquidity covary with each other and with seller private information.

My model builds on the retention signaling framework of DeMarzo and Duffie (1999) by adding competitive search (Guerrieri, Shimer, and Wright 2010; Wright et al. 2019). A continuum of sellers each hold one unit of a divisible asset, and each seller has private information about his asset's future payoff. Sellers are less patient than buyers, which creates gains from trade. Buyers pay a fixed cost to post prices and quantities at which they are willing to trade with sellers, where each price-quantity pair defines a submarket. Sellers select the most profitable submarket at which to attempt trade. Search frictions imply trade is not guaranteed for buyers and sellers; agents form rational expectations about the equilibrium ratio of buyers to sellers in each market, internalizing that a higher ratio implies a higher probability of sale for sellers and a lower probability of purchase for buyers. I refer to the seller's equilibrium probability of sale as *liquidity* and their asset quantity *not* offered as *retention*.

In a benchmark setting, I adopt standard assumptions from the competitive search literature and assume that sellers have common and publicly known patience, so only asset quality is private information. In this setting, I show that there is only one equilibrium, and it features no retention. Instead, separation is achieved by giving high quality sellers low liquidity and a compensating higher price. Intuitively, screening sellers with low liquidity is less costly than screening with low quantity, because it requires fewer buyers paying participation costs, which are dead weight loss. This result echoes the two-type example of Application III in Guerrieri, Shimer, and Wright (2010), in which sellers are screened with matching probability rather than ex-post trading probability conditional on a match.

I shed light on this result and further illuminate the relationship between retention and liquidity by analyzing two generalizations of the benchmark setting. First, I allow buyer entry costs to depend on the quantity they post. This sharpens the analogy between retention and liquidity: just as higher liquidity is costly because it requires more buyers paying entry costs, so also higher quantity may be costly because it may require higher entry costs per buyer.

In this generalized setting, I present sufficient conditions that characterize which screening instrument is used in equilibrium. If the inverse elasticity of the matching function is higher than the elasticity of the entry cost function, there is no retention and sellers are screened with liquidity, as in the benchmark setting. Intuitively, screening sellers with low liquidity is less costly than with low quantity, because the reduction in the number of buyers in the market more than offsets the rise in per-buyer entry costs due to high quantity. Conversely, if the inverse elasticity of the matching function is higher than the elasticity of the entry cost function, there is full liquidity and sellers are screened by low quantity instead. If the two elasticities are equal, the two instruments are perfect substitutes, so multiple equilibria exist.

In the second generalization, I expand the space of private information to include not only asset quality but also seller patience. It is natural to consider quality and patience together. For example, in addition to being unable to directly observe the profitability of a subsidiary, potential acquirers may also be unable to observe how credit constrained the selling parent firm is, which implies how urgently the parent requires cash proceeds from the sale.

In this expanded setting, I show that there exists an equilibrium in which retention and liquidity work together to fully separate both dimensions of private information. Sellers of higher private value charge a higher price but sell a lower *expected* quantity of their asset, thus preventing mimicry by lower value sellers. So buyers can identify a seller's private value by observing the price that they select. However, because patience is heterogeneous, sellers of the same private value may have assets of different quality. Among sellers of common value, those with higher quality assets offer a lower quantity but are compensated with higher liquidity. So buyers can further identify a seller's asset quality by observing the quantity in addition to the price.

This equilibrium delivers several new predictions about the price, quantity, and liquidity which do not require the difficult task of measuring private information. The key novelty is conditional dispersion: conditional on any one of the three variables, there is negatively correlated dispersion in the other two. For example, for a fixed price, quantity and liquidity are negatively correlated. Intuitively, sellers of common private value have identical incentives and must receive the same price and sell the same expected quantity, so higher liquidity must be offset by lower quantity in equilibrium.

Some empirical papers have found creative ways to measure seller private information; for settings where this is possible, the paper delivers novel predictions about how private information varies with quantity and liquidity. For a fixed price, asset quality correlates positively with liquidity but negatively with quantity. In addition, for fixed seller patience, liquidity may be hump-shaped in asset quality, a striking departure from the monotonic predictions of prior competitive search papers.

Finally, I study the effects of a rise in the relative mass of impatient sellers. Consistent with prior literature, average price falls and total volume rises, as in a fire sale. However, the equilibrium's conditional dispersion leads to new results: conditional on the price, average quality rises, which leads to rising average liquidity and falling average quantity. The force underlying these new results—impatient sellers are more willing to sell good assets—predicts a rising price in the pooling equilibria of Eisfeldt (2004) and Uhlig (2010), which they emphasize is counterfactual. But with full separation of two dimensions, this same force instead predicts the novel price-conditional effects described above.

Relation to Literature. Leland and Pyle (1977) showed how retention may function as a credible signal of asset quality. This insight has been applied in many models, [1] and receives empirical support in the markets for syndicated loans, mortgage-backed securities, and crowdfunding.[2] I contribute to this literature by introducing search frictions, showing how retention interacts with liquidity.

On the other hand, the liquidity channel is illustrated by Guerrieri, Shimer, and Wright (2010) and Chang (2018), who combine competitive search with adverse selection and find an equilibrium in which liquidity is decreasing in asset quality.[3] Chang (2018) and Guerrieri

1. DeMarzo and Duffie (1999); DeMarzo (2005); DeMarzo, Frankel, and Jin (2015); Grinblatt and Hwang (1989); He (2009); Hartman-Glaser (2012); Chari, Shourideh, and Zetlin-Jones (2014)

2. Ivashina (2009), Demiroglu and James (2012), Begley and Purnanandam (2016), Hildebrand, Puri, and Rocholl (2017), and Brogaard, Ngo, and Xia (2018).

3. Probability of sale in my static setting is a simplified representation of speed of sale; several dynamic papers explicitly study selling speed, such as Chang (2018), Guerrieri and Shimer (2014), and Fuchs and Skrzypacz (2012), Daley and Green (2012), and Kremer and Skrzypacz (2007). Of these, only Chang (2018) studies private patience, and none jointly study quantity screening.

and Shimer (2018) both explore multidimensional private information, but neither allow for multiple screening instruments, and hence focus on pooling equilibria. In Li (2018), informed sellers design securities, which implies a large number of signals but only one dimension of private information.[4] Hong and Rady (2002) study how informed traders adjust the quantity of their trades when uncertain about exogenous liquidity. I contribute to this literature by studying the interaction of endogenous liquidity with retention in an equilibrium which fully separates multiple dimensions, yielding novel predictions about both instruments.

In related work, Kurlat (2016) studies a setting of competitive trade with adverse selection, in which buyers are heterogeneously informed. He assumes that sellers may list the same fraction of an asset in multiple markets (double-listing), so neither retention nor liquidity are effective screening instruments, and a pooling equilibrium results. In Li (2018), sellers may sell different fractions of the same asset in multiple markets, but cannot double-list the same fraction. As a result, retention is not credible, but separation is sustained via liquidity instead. I follow most of the competitive search literature by assuming exclusive trade in a single market, so both liquidity and retention are credible screening instruments that sustain separation.

The paper's main technical contribution is to find an equilibrium which fully separates two-dimensional types using two screening instruments. Many related multidimensional papers either do not characterize full separation[5] or require linearity (Quinzii and Rochet 1985) or symmetry (He 2009; DeMarzo, Frankel, and Jin 2015) which does not apply in my setting. Because agents on each side of the market care directly about only one dimension of private information, the seller's incentive compatibility constraint collapses to a single dimension, allowing me to characterize the equilibrium with an ordinary differential equation.[6]

4. See also Viswanathan (1987), who studies the firm's choice among multiple signaling instruments in a dynamic model where managers have private information about the firm's future prospects.

5. See Engers (1987), Rochet and Choné (1998), Edmans and Mann (2016), Chang (2018), and Guerrieri and Shimer (2018).

6. In Grinblatt and Hwang (1989), uninformed buyers care about only one dimension, which also simplifies their characterization of two-dimensional separation.

2 Benchmark Setting

I begin by presenting a benchmark setting which embeds the retention model of DeMarzo and Duffie (1999) in a standard competitive search framework (Wright et al. 2019; Guerrieri, Shimer, and Wright 2010; Chang 2018).

The environment consists of a unit mass continuum of risk-neutral sellers and an endogenous mass continuum of risk-neutral buyers. The model lasts for one period, which begins at date 0 and ends at date 1. Each seller owns one unit of a divisible asset that generates future cash flows $f \in [\underline{f}, \bar{f}] \subset \mathbb{R}_{++}$. The quality f of each asset is privately known by its owner. Each seller discounts future cash flows with common, publicly known factor $\delta \in (0, 1)$, so the seller's value of retaining the assets is therefore δf; this discounting gives the seller incentive to raise cash by selling some portion of the asset.[7] The probability density function of seller types is given by $g : [\underline{f}, \bar{f}] \to \mathbb{R}_{++}$.

There is a large set of homogeneous buyers, each with discount factor normalized to 1. Each buyer may or may not participate in the market: they can enter, which provides a buyer an opportunity to match with a seller, if and only if they pay fixed cost $k > 0$. When I say a buyer enters the market, I mean that he posts a contract (p, q) which specifies a quantity q of the asset to be traded and a price p per unit of the asset. By posting I mean that the buyer announces and commits to the contract, and all sellers can see what every buyer posts. To keep the analysis focused, in this project, I consider an environment where buyers and sellers have a single opportunity to match, and matching is bilateral. If a buyer and a seller match, the pair enter into a relationship described by the contract.

I now turn to the matching process. There is a continuum of submarkets, and each posted contract (p, q) is traded in a separate submarket. Each seller observes what all the buyers post and then directs his search to any contract (i.e., submarket) he likes. Within each submarket, sellers search for a buyer, and matching is bilateral, so at most one seller

7. All results go through if the seller instead suffers an additive holding cost c for all unsold units of the asset, as in Chang (2018). See Section C of the online appendix.

ever contacts each buyer. As explained below, positive entry costs k imply a finite measure of buyers in each submarket. Let $\theta(p, q)$ denote the buyer-seller ratio, or market tightness, associated with a contract (p, q), defined as the measure of buyers posting (p, q) divided by the measure of sellers seeking (p, q), $\theta : \mathbb{R}_+ \times [0, 1] \to [0, \infty]$.[8]

A seller who directs his search to (p, q) matches with a buyer with probability $m(\theta(p, q))$, independent of type, where the matching function $m : [0, \infty] \to [0, 1]$ is nondecreasing. Otherwise the seller is unmatched. A buyer offering (p, q) matches with a seller with probability $n(\theta(p, q))$, where $n : [0, \infty] \to [0, 1]$ is nonincreasing, and otherwise is unmatched. I impose $m(\theta) = \theta n(\theta)$ for all θ, since the left hand side is the matching probability of a seller and the right hand side is the matching probability of a buyer times the buyer-seller ratio. Together with the monotonicity of m and n, this implies both functions are continuous. I make the standard assumptions[9] that $m(\theta)$ is strictly increasing, strictly concave, twice continuously differentiable, and $0 \leq m(\theta) \leq \min[\theta, 1]$, which guarantees that both m and n are in $(0, 1)$ for all $\theta > 0$. I also assume that $\lim_{\theta\to\infty} m(\theta) = \lim_{\theta\to 0} m(\theta)/\theta = 1$; this implies $\lim_{\theta\to\infty} n(\theta) = \lim_{\theta\to\infty} m(\theta)/\theta \to 0$, so that free entry and positive entry costs $k > 0$ imply a finite measure of buyers in each submarket. Because one is an increasing function of the other, I refer to both the seller's matching probability $m(\theta)$ and the market tightness θ as *liquidity* throughout the paper.

I assume that regardless of seller type, ex-post gains from trade exceed the participation costs: $(1 - \delta)f > k$. Without this assumption, there may exist some sellers which cannot attract any buyers, regardless of the terms of trade.

8. The equilibria below feature a continuum of submarkets, implying zero measure of buyers and sellers in each submarket. But if $\beta(p, q)$ and $\sigma(p, q)$ denote the measure of buyers and sellers, respectively, posting contracts less than or equal to (p, q), the market tightness function $\theta(p, q)$ is still well-defined by the Radon-Nikodym derivative $d\beta(p, q)/d\sigma(p, q)$. See Section 6 of Wright et al. (2019).

9. See the first example of Guerrieri, Shimer, and Wright (2010), as well as Eeckhout and Kircher (2010). For example, under the urn-ball matching technology, $m(\theta) = 1 - e^{-\theta}$, as in Rogerson, Shimer, and Wright (2005). As Eeckhout and Kircher (2010) point out, because $m(\cdot)$ is strictly increasing and bounded, strict concavity has to hold for some θ, so they "make the standard assumption that this property extends to the entire domain."

2.1 Equilibrium

Both buyers and sellers have rational expectations about the equilibrium market tightness $\theta(p,q)$ associated with each pair (p,q); agents on both sides take the equilibrium function $\theta(p,q)$ as given when choosing terms that maximize their own expected payoffs. Buyers also form rational beliefs μ about the composition of sellers in each market, where $\mu(f|p,q)$ denotes the cumulative distribution of seller's types in market (p,q).

Definition 1. An equilibrium is a set $M \subset \mathbb{R}_+ \times [0,1]$ of price-quantity pairs (p,q), a market tightness function $\theta(p,q) : \mathbb{R}_+ \times [0,1] \to \mathbb{R}_+$, and buyer beliefs $\mu(f|p,q) : [\underline{f}, \bar{f}] \times \mathbb{R}_+ \times [0,1] \to [0,1]$ satisfying:

(i) Seller optimality: For all $(p,q) \in M$, if f is in the support of $\mu(\cdot|p,q)$, then

$$(p,q) \in \underset{(p',q') \in M}{\arg\max}\, m(\theta(p',q'))q'(p' - \delta f).$$

(ii) Buyer optimality:

(a) Zero profit: For all $(p,q) \in M$, $\theta(p,q) > 0$ and

$$\frac{m(\theta(p,q))}{\theta(p,q)} q(E[\tilde{f}|p,q] - p) - k = 0$$

(b) No profitable deviation: For all $(p,q) \notin M$,

$$\frac{m(\theta(p,q))}{\theta(p,q)} q(E[\tilde{f}|p,q] - p) - k \leq 0.$$

Condition (i) states that if buyers expect a certain type for a certain pair of terms, that type must find it optimal to select those terms. Sellers internalize that their chosen terms (p,q) imply a certain market tightness θ, which in turn implies a certain probability of sale $m(\theta)$. Contingent on a sale, sellers exchange quantity q of their asset, receiving price p per unit but giving up private value δf per unit. Condition (ii)(a) states that buyers must break even in equilibrium. Buyers' objective function shows that they must pay cost k to search for sellers regardless of whether they find any. Contingent on a match, buyers receive q units of the asset, which is worth $E[\tilde{f}|p,q]$ per unit in exchange for price p per unit. Condition (ii)(b) stipulates that buyers cannot profit from posting off-equilibrium (p,q); I use the standard belief refinement, which states that if buyers post off-equilibrium (p,q), they expect the types that are willing to accept the lowest market tightness.[10]

In this benchmark setting, because illiquidity is essentially a probabilistic form of retention, one might suppose that the two screening instruments would be equivalent, so multiple equilibria might exist. The following proposition shows that this is not the case.

Proposition 1 (No Retention Benchmark)**.** *If private information consists only of the single dimension asset quality $f \in [\underline{f}, \bar{f}]$, then a unique equilibrium exists; there is no retention, so $q(f) = 1$ for all f, liquidity $\theta(f)$ is strictly decreasing in quality f, and price $p(f)$ is strictly increasing in quality f.*

The unique equilibrium is the least-cost fully separating equilibrium, and it features no retention. Instead, high quality sellers suffer lower liquidity to prevent mimicry, and are compensated by a higher price. Although liquidity $m(\theta)$ and quantity q may seem interchangeable, a higher market tightness θ requires more buyers paying entry costs k, which is deadweight loss. So screening sellers with low liquidity θ is more efficient than low quantity q, because it requires less costly entry by buyers. A similar result for the case of two types is found in Result 4 of Guerrieri, Shimer, and Wright (2010), where ex-post trading probability in their setting is equivalent to quantity in my setting. I present this result here merely as a benchmark against which later results will be compared.

10. See Definition A.1 in the Online Appendix for a precise statement of this refinement, and Guerrieri, Shimer, and Wright (2010) and Chang (2018) for a detailed motivation.

3 Quantity-Dependent Entry Costs

In this section, I sharpen the analogy between retention and liquidity by relaxing standard assumptions on the entry costs k. I allow participation costs k to depend on the quantity q posted, and assume these costs are mild enough that all types can feasibly trade: for all f, there exists q such that $q(1-\delta)f > k(q)$. In addition, I relax the requirement that m is strictly increasing and strictly concave everywhere; letting $\bar{\theta} \equiv \inf\{\theta \geq 0 : m(\theta) = 1\}$, I allow that m may attain 1 for some finite $\bar{\theta}$, and let h denote the inverse of m over $[0, \bar{\theta})$, so that for any liquidity $y \equiv m(\theta)$, $h(y) = \theta$.

With these generalizations and notation, expressing each seller's welfare in terms of the screening instruments (y, q) alone illuminates the welfare cost of each instrument. Using buyer zero-profit to eliminate price p, a seller with asset quality f receives expected profit

$$yq(1-\delta)f - h(y)k(q)$$

in any separating equilibrium. This expression makes the cost of each screening instrument clear: just as liquidity y requires $h(y)$ buyers per seller (extensive costs), so does quantity q require $k(q)$ entry costs per buyer (intensive costs). The following theorem shows that the instrument with higher cost elasticity $\epsilon(\cdot)$ is the one used in equilibrium. I defer to the proofs the particular functional forms of each equilibrium.

Theorem 1. *(i) (Liquidity Screening) If $\epsilon_h(y) > \epsilon_k(q)$ for all $(y, q) \in [0,1]^2$, then a unique equilibrium exists; there is no retention, so $q(f) = 1$, and liquidity separates all types.*

(ii) (Quantity Screening) If $\epsilon_h(y) < \epsilon_k(q)$ for all $(y, q) \in [0,1]^2$, then $\bar{\theta} < \infty$ and a unique equilibrium exists; there is no illiquidity, so $m(\theta(f)) = m(\bar{\theta}) = 1$, and quantity q separates all types.

(iii) (Perfect Substitutes) If $\epsilon_h(y) = \epsilon_k(q)$ for all $(y, q) \in [0,1]^2$, then there exists a strictly decreasing function $\phi(\cdot)$ such that any allocation satisfying $m(\theta(f))q(f) = \phi(f)$ and

$\theta(f) \leq \bar{\theta} = 1$ for all $f \in [\underline{f}, \bar{f}]$ is an equilibrium.

Item (i) nests the case of constant k where $\epsilon_k = 0$ as in Proposition 1 and Application III of Guerrieri, Shimer, and Wright (2010). Because the unique equilibrium is least-cost separating, it is the allocation in which the expected fraction sold yq in each submarket is just low enough to prevent mimicry by lower types, while miminizing the posting costs $h(y)k(q)$. If $\epsilon_h > \epsilon_k$, then by reducing probability y and raising quantity q so that expected fraction yq is unchanged, the reduction in buyers-per-seller $h(y)$ more than offsets the rise in per-buyer posting costs $k(q)$. So screening sellers with a low matching probability $y = m(\theta)$ is less costly than screening with low quantity q.

The intuition is similar for items (ii) and (iii). Interestingly, item (ii) shows that if entry costs $k(\cdot)$ are sufficiently elastic, then retention may emerge as the optimal screening instrument, for example if $m(\theta) = \min\{\theta, 1\}$ and $k(q) = q^2$. And item (iii) shows that multiple equilibria may exist, for example if $m(\theta) = \min\{\theta, 1\}$ and $k(q) = cq$ for some constant c.[11] In this case, liquidity and quantity are equivalent screening instruments, and can take any values as long as the expected fraction $m(\theta)q$ follows the separating schedule $\phi(\cdot)$. Finally, note that although the theorem covers a large class of functions $k(\cdot)$ and $h(\cdot)$, it is agnostic about functions for which the elasticities are not ranked uniformly for all $(y, q) \in [0, 1]^2$.

4 Private Quality and Patience

The previous section departed from the standard assumptions in the benchmark setting by letting entry costs scale with quantity. This section returns to the benchmark assumption of constant entry costs k, but departs from the benchmark setting in a different way: I assume that sellers differ not only in asset quality $f \in [\underline{f}, \bar{f}]$, but also in patience $\delta \in [\underline{\delta}, \bar{\delta}]$, and that both dimensions are privately known to the seller. The equilibrium definition is unchanged

11. The proof shows these are the only functions satisfying $\epsilon_h(y) = \epsilon_k(q)$ for all $(y, q) \in [0, 1]^2$ and $\lim_{\theta \to 0} n(\theta) = 1$.

except that buyers form beliefs about seller patience δ in addition to asset quality f.

The solution is simplified if I transform the type space so that the first dimension is asset quality f and the second dimension is the seller's discounted asset valuation $v \equiv \delta f$. Since there is a one-to-one mapping between the original and transformed type space, this transformation is without loss of generality.

There may exist multiple equilibria in this setting, but I do not attempt an exhaustive characterization of them. Instead, I focus on the fully separating equilibrium, mainly because it is robust to off-equilibrium beliefs in a way that other equilibria are not. For example, the online appendix analyzes a retention-free equilibrium which separates only up to $v = \delta f$, similar to Section III of Guerrieri and Shimer (2018). Because I assume that entry costs k are constant, retention is a less efficient screening instrument than liquidity (consistent with Theorem 1), so the retention-free equilibrium Pareto dominates the fully separating equilibrium analyzed here. However, the retention-free equilibrium requires additional belief restrictions that Guerrieri and Shimer (2018) emphasize are quite strong and without sound theoretical justification; no such additional restrictions are required for the fully separating equilibrium. Furthermore, the structure and intuition of the retention-free equilibrium is nearly identical to that of the benchmark in Proposition 1; in contrast, the following equilibrium sheds new light on the relationship between retention and liquidity by showing how they may work together to separate both dimensions.

Theorem 2 (Two-Dimensional Separation)**.** *The class of equilibria which fully separate both quality f and patience δ contains one and only one equilibrium. In this equilibrium:*

(i) sellers of common value v receive the same expected profit, sell the same expected quantity $m(\theta)q$, and receive the same price p;

(ii) as seller value v increases, price p strictly increases, expected quantity sold $m(\theta)q$ strictly decreases, and given asset quality f, liquidity $\theta(f, v)$ strictly decreases;

(iii) given seller value v, liquidity $\theta(f, v)$ strictly increases in asset quality f, and quantity

sold $q(f,v)$ *strictly decreases in asset quality* f*.*

Separation is achieved in two layers: buyers identify a seller's private value v by observing which price p the seller selects (part ii), and having identified v, buyers identify the asset quality f by observing the selected quantity q (part iii).

Part (i) of the theorem states that if two sellers (f,v) and (f',v') have assets of different quality ($f \neq f'$), but value their assets the same ($v = v'$), they must receive the same profit. If not, then the lower profit seller would always pretend to be the higher profit seller, and there would be no way to punish him for doing so, because he has the same incentives as the higher profit seller. This constant profit is achieved by giving them the same expected fraction $m(\theta)q$ and price p.

Part (ii) follows because single-crossing holds along the dimension v. Intuitively, for a fixed price p and expected quantity sold $m(\theta)q$, higher value sellers receive a lower premium $p - v$ from each unit sold, so they value trade less. Accordingly, they can be separated with a lower expected quantity $m(\theta)q$ as long as they are compensated with a higher price p. Liquidity $\tilde{\theta}(f,v)$ is determined by the attractiveness (to the buyer) of the trade terms relative to the expected quality of the asset. Buyers find the higher price and low expected fraction unattractive, so the ratio θ of buyers to sellers falls as seller value v increases.

Part (iii) follows because the common price among sellers with common value v implies that those with higher quality assets sell them at a larger discount $f - p$. Buyers are attracted to their deeper discount, driving up the buyer seller ratio θ and therefore the probability of sale $m(\theta)$. However, in equilibrium, sellers of common private value v not only receive the same price p, but also sell the same expected quantity $m(\theta)q$, so quantity q decreases as asset quality f increases in order to offset the increasing sale probability $m(\theta)$. Part (iii) is illustrated in Figure 1.

Remark 1. Retention exists in the two-dimensional equilibrium of Theorem 2, whereas in the one dimensional benchmark setting of Proposition 1, any allocation featuring retention ($q < 1$) cannot be an equilibrium. This is because in the benchmark setting, any equilibrium

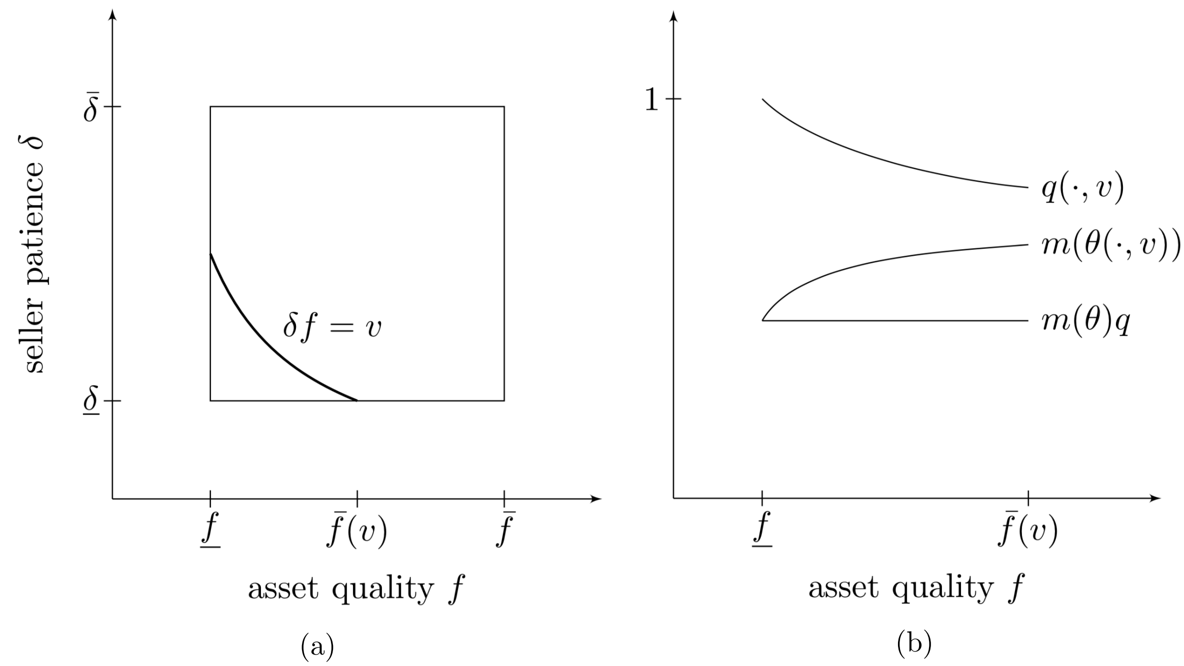

Figure 1: The curve in panel 1a traces out a set of sellers with common value v. Panel 1b shows the cross section of quantity sold q and liquidity $m(\theta)$ along this curve, illustrating part (iii) of Theorem 2.

price (seller value δf) is associated with a unique asset quality f, so buyers would find if profitable to deviate by offering to buy a higher off-path quantity $q' > q$. But under two-dimensional separation, any equilibrium price p (seller value δf) is associated with a variety of qualities f; because lower quantities q are associated with higher quality f, buyers are indifferent across a range of quantities q.

Remark 2. The form of the matching function m is important for full separation. A strictly increasing matching function is sufficient for separation of f, so that retaining sellers can always be compensated with higher liquidity. A differentiable matching function is sufficient for separation of both f and v, as kinks may imply that distinct types with the worst asset quality $\underline{f}$ have the same liquidity θ and price p. While these two assumptions are standard in the literature, some papers use the piecewise linear function $m(\theta) = \min\{\theta, 1\}$, which violates both; in that case, full separation cannot be achieved, but there does exist an equilibrium in which a subset of types reveal both dimensions via price and quantity, in a manner similar to Theorem 2.[12]

5 Implications

By distinguishing quantity from liquidity, the separating equilibrium makes a number of novel predictions about how these values covary with each other, with the price, and with seller private information. For example, in the context of IPOs, quantity can be measured by the size of the issue, and liquidity can be measured by the fraction of offered shares which are subscribed by investors (Amihud, Hauser, and Kirsh 2003). In the context of syndicated loans, quantity may be measured by the fraction of the loan offered by the lead bank to participant banks, and illiquidity may be measured by the pipeline risk (undersubscription) of participant banks (Bruche, Malherbe, and Meisenzahl 2017) or by the time it takes to form a syndicate (Godlewski 2010).[13]

5.1 Trade Terms and Liquidity

The model delivers several testable predictions on p, q, and θ which do not require the difficult task of measuring private information. The key novelty is conditional dispersion: conditional on any one of the three variables, there is dispersion and in fact negative correlation between the other two.

Proposition 2. *In the unique separating equilibrium characterized in Theorem 2,*

(i) Given p, variables q and θ are negatively correlated.

(ii) Given q, variables p and θ are negatively correlated.

(iii) Given θ, variables p and q are negatively correlated.

12. Section B of the online appendix gives a detailed explanation of the remark, as well as a formal presentation of this equilibrium.

13. Liquidity $m(\theta)$ may be interpreted not only as the probability of sale within a given time period, but also as a reduced form representation of the time it takes to sell the asset. This dynamic interpretation is also used in Guerrieri, Shimer, and Wright 2010, Eeckhout and Kircher 2010.

Relative to previous literature, the most novel prediction is item (i), which follows from part (iii) of Theorem 2. For a fixed price p, quantity q falls to offset rising θ in order to keep expected quantity $m(\theta)q$ constant across sellers of common value. Godlewski (2010) shows that if lead arrangers of syndicated loans retain a larger share (sell a low quantity), the duration of the syndication process is shortened (liquidity is high), indicating that quantity and liquidity are unconditionally negatively correlated; this is consistent with my prediction, but I know of no papers which have tested the correlation of quantity with liquidity *conditional* on the price.

Items (ii) and (iii), which follow from part (ii) of Theorem 2, extend results of prior literature. For example, Guerrieri, Shimer, and Wright (2010) and Chang (2018) feature item (ii) while assuming that $q = 1$, and DeMarzo and Duffie (1999) features item (iii) while assuming $m(\theta) = 1$. In contrast, the proposition shows that these correlations hold when conditioning on any endogenous value of liquidity or quantity. Empirical work shows that retention is associated with higher prices (Begley and Purnanandam 2016) or equivalently lower required return (Ivashina 2009), but they do not condition on liquidity. To my knowledge, these conditional predictions have not been tested.

5.2 Private Information

In settings where seller private information can be measured,[14] the model delivers several novel predictions regarding the way in which such information varies with quantity and liquidity. The second result requires additional structure on the matching function, so I assume constant elasticity of substitution for simplicity.

Proposition 3. *In the unique separating equilibrium characterized in Theorem 2:*

(i) given price p, asset quality f positively correlates with liquidity θ, and negatively cor-

14. The empirical literature on retention in the introduction, as well as Adelino, Gerardi, and Hartman-Glaser (2019), proxy for asset quality by conditioning on buyer observables and measuring ex-post asset performance. Annual 10-K filings may retroactively identify financial contraints (Kaplan and Zingales 1997) and thus proxy for seller impatience.

relates with quantity q;

(ii) if $m(\theta) = (1 + \theta^{-r})^{-1/r}$ and r is sufficiently high, then for any patience δ, liquidity $\theta(f, \delta)$ is strictly increasing in f for all $f < \bar{\delta}\underline{f}/\delta$ and strictly decreasing in f for all $f > \bar{\delta}\underline{f}/\delta$.

Because sellers of common value receive the same price, part (i) follows from Theorem 2, part (iii). Bruche, Malherbe, and Meisenzahl (2017) document that less subscribed syndicated loan offerings tend to be associated with less underpricing, consistent with my prediction in (i) that for a fixed price, quality (underpricing) is increasing in liquidity (subscription). Keloharju (1993) studies 80 cases of IPOs in Finland during 1984-1989 and finds that the initial return is a declining function of buyer order size. This is consistent with my prediction that, holding the price fixed, quantity is decreasing in quality.

Part (ii) predicts that liquidity may be hump-shaped in asset quality, a striking contrast to prior literature: if patience is public, as in Guerrieri, Shimer, and Wright (2010) and Proposition 1, or retention is ruled out by assumption, as in Chang (2018) and Guerrieri and Shimer (2018), then liquidity is monotonically decreasing. Intuitively, holding seller patience δ fixed, an increase in asset quality f raises the seller value v and buyer value f simultaneously, and each channel affects liquidity θ differently. By parts (ii) and (iii) of Theorem 2, the two effects are respectively negative and positive; the first effect dominates only for high-quality assets.[15]

Empirical work[16] shows that higher quality assets may be sold with a lower probability or sold later, but the specifications in these papers cannot detect my hump-shaped prediction even if it were present.[17] To my knowledge, this prediction has not been tested.

15. Section D of the online appendix gives detailed intuition for this result.

16. Downing, Jaffee, and Wallace (2009), Keys et al. (2010), Krainer and Laderman (2014), Li (2018), and Adelino, Gerardi, and Hartman-Glaser (2019)

17. The nonparametric specification of Adelino, Gerardi, and Hartman-Glaser (2019) finds that asset quality is somewhat hump-shaped in time-to-sale, but their specification does not allow detection that selling speed is hump-shaped in asset quality.

5.3 Fire Sales

Finally, I study fire sales by considering a shock to seller patience.[18] I assume that the support remains fixed, but the new marginal distribution $\hat{g}(\delta)$ is dominated by the original $g(\delta)$ in the sense of the monotone likelihood ratio property, so that the relative mass of impatient sellers rises. The following proposition first echoes prior literature by showing that features of a fire sale may occur, and then goes on to identify new price-conditional predictions.

Proposition 4 (Fire Sales)**.** *Suppose that f and δ are independent. If the type density $g(f, \delta) = g(f)g(\delta)$ is replaced by $\hat{g}(f, \delta) = g(f)\hat{g}(\delta)$, where $g(\delta)/\hat{g}(\delta)$ is strictly increasing in δ, then*

(i) average[19] *price p falls, total volume rises, and average liquidity θ rises.*

Furthermore, conditional on the price p,

(ii) average quality f rises, average quantity q falls, and average liquidity θ rises.

As in a fire sale, average price falls, total volume rises and average liquidity rises; this result is also found in Chang (2018), who cites evidence of fire sales with large distress discounts and high volume. This is because the relative mass of impatient sellers rises, and they are willing to offer deep discounts in exchange for selling a high expected quantity, which attracts buyers and therefore liquidity.

The price-conditional dispersion of the equilibrium gives rise to the new fire sale predictions in part (ii), which to my knowledge have not been tested. For a fixed price, average quality rises, because for that price it is the impatient sellers who are willing to sell good assets. In one-price pooling equilibria (e.g., Eisfeldt 2004, Uhlig 2010, Kurlat 2016), where price is determined by average quality, this force may cause the price to rise. But in this

18. This notion of fire sales is in the spirit of Kurlat (2016), Uhlig (2010), and Chang (2018). In contrast, Guerrieri and Shimer (2014) model fire sales as the onset of private information about quality.

19. For ease of exposition, averages are computed per offering seller. The online appendix gives some related predictions which weight the average by quantity and condition on successful sale.

fully separating equilibrium, where price is determined by the worst asset among sellers of common value, the price received by these sellers remains fixed as the average quality rises. As a result, average liquidity rises as well, which is offset by falling average quantity to prevent mimicry.

6 Conclusion

The paper develops a unified framework in which both retention and liquidity can screen sellers. Here, the notion of liquidity is search-theoretic, a natural definition which draws on recent advances in the literature. I present general conditions on the matching function and entry costs which characterize which instrument will screen sellers in equilibrium. In addition, I show that if private information includes not only asset quality but also seller impatience, there exists an equilibrium in which both liquidity and retention work together to fully separate both dimensions of private information.

This equilibrium leads to several novel implications. For example, the equilibrium predicts price-conditional dispersion in quantity, liquidity, and quality. In contrast to competitive search models with fewer screening instruments or dimensions of private information, liquidity may be hump-shaped in asset quality. Furthermore, if the relative mass of impatient sellers rises, then not only does the equilibrium predict a fall in price and a rise in volume as in a fire sale, but it also predicts that conditional on the price, average quality and liquidity rise, while average quantity falls.

Proofs

Throughout, lemmas whose numbers are preceded by the letter A are stated and proven in Section A of the online appendix.

Proof of Proposition 1: Except for uniqueness, the proposition is a special case of Theorem 2, where $\underline{\delta} = \bar{\delta}$. The online appendix gives a self-contained proof.

Proof of Theorem 1: The proof of parts (i) and (ii) is the same as for Proposition 1, except for Lemma A.10 (as well as the second part of the proof of Lemma A.9, which is basically the same). To prove the analogue of Lemma A.10, take some IC separating allocation $(p, q, \theta)(f)$ that satisfies buyer zero profit. Let $h(\cdot)$ be the inverse of the invertible part of m, so that $h(y) = \theta$ and $m(\theta) = y$, and write the buyer zero profit condition as

$$yq(f - p) = k(q)h(y). \tag{1}$$

If buyers hold price $p(f)$ fixed, but deviate by offering $q \neq q(f)$, then by Lemma A.5 they expect type f and expect θ which satisfies $m(\theta)q = m(\theta(f))q(f) \equiv C$; in terms of h and y, buyers expect market tightness $h(y)$ such that $yq = C$, which implies $\ln y + \ln q = \ln C$. So for a small change in q, we have $d \ln q = -d \ln y$, the left-hand side of (1) is fixed, and the elasticity of the right-hand side is:

$$\frac{d \ln[k(q)h(y)]}{d \ln q} = \frac{d \ln k(q)}{d \ln q} + \frac{d \ln h(y)}{d \ln q} = \frac{d \ln k(q)}{d \ln q} - \frac{d \ln h(y)}{d \ln y} = \epsilon_k(q) - \epsilon_h(y). \tag{2}$$

Part (i). Suppose to the contrary that in equilibrium, there exists a type f which receives $q(f) < 1$. If $\epsilon_h(y) > \epsilon_k(q)$ for all $(y, q) \in [0, 1]^2$, then (2) is negative, so a higher q reduces the right-hand side of (1) and is therefore a profitable deviation, contradicting equilibrium. Applying arguments identical to Proposition 1 shows that the only equilibrium takes the same form as in Proposition 1, but with $k(1)$ in place of constant k.

Part (ii). Suppose there exists a type f which receives $\theta(f) < \bar{\theta} < \infty$ (see Lemma A.11).

Then $y(f) \equiv m(\theta(f)) < 1$. If $\epsilon_h(y) < \epsilon_k(q)$ for all $(y, q) \in [0, 1]^2$, then (2) is positive, so a lower q reduces the right-hand side of (1) and is therefore a profitable deviation, contradicting equilibrium. So for all $f \in [\underline{f}, \bar{f}]$, $\theta(f) = \bar{\theta}$. Applying identical arguments as in Proposition 1 shows that the only equilibrium must have the form $\theta(f) = \bar{\theta}$, $p(f) = f - \frac{k(q(f))}{n(\bar{\theta})q(f)}$ and $q(f)$ as the decreasing solution to $q(f) + [(1-\delta)f - \bar{\theta}k'(q(f))]q'(f) = 0$, where $q(\underline{f}) = q_{CI}(\underline{f})$.

Part (iii). Lemma A.12 shows that there exists a constant c such that if $yq = \phi$, then $k(q)h(y) = c\phi$. I claim that any allocation $(p(f), \theta(f), q(f))$ satisfying $p(f) = f - c$, $\theta(f) \leq \bar{\theta}$, and $m(\theta(f))q(f) = \phi(f)$, where $\phi(\cdot)$ is the decreasing solution to

$$\phi(f) + \Big((1-\delta)f - c\Big)\phi'(f) = 0 \qquad \phi(\underline{f}) = m(\theta_{CI}(\underline{f}))q_{CI}(\underline{f})), \tag{3}$$

is an equilibrium.

To show that seller optimality is satisfied, I show that the allocation $(p(f), \theta(f), q(f))$ is a mechanism that satisfies GIC. The solution $\phi(\cdot)$ to the ODE must be differentiable and therefore continuous, so $\Pi(f) = m(\theta(f))q(f)(p(f) - \delta f) = \phi(f)(f - c - \delta f)$ is continuous. Differentiating the above expression for $\Pi(\cdot)$ and applying (3) gives $\Pi'(f) = \phi'(f)[(1-\delta)f - c] + \phi(f)(1-\delta) = -\delta\phi(f) = -\delta m(\theta(f))q(f)$, and because $\phi(f) = m(\theta(f))q(f)$ is decreasing, Lemma A.3 ensures that GIC is satisfied.

Because $p(f) = f - c = f - k(q)h(y)/(qy)$, the allocation satisfies buyer zero-profit (1). To show that off-equilibrium (p, q) are not profitable, suppose that buyers post (p', q'), where there exists some f such that $p' = p(f)$, but $q' \neq q(f)$. Then because $\epsilon_h = \epsilon_k$, Equation (2) implies that any change in q will leave the right-hand side of (1) unchanged, so the deviation will not strictly profit buyers. The argument that $p' \notin [p(\underline{f}), p(\bar{f})]$ is not a profitable deviation is similar to Theorem 1. □

Proof of Theorem 2: I let $S \equiv [\underline{f}, \bar{f}] \times [\underline{\delta}, \bar{\delta}]$, $\tilde{S} \equiv \{(f, v) : v = \delta f, (f, \delta) \in S\}$, and $V \equiv \{\delta f : (f, \delta) \in S\}$. To establish uniqueness, I show that the equilibrium definition along with full separation implies the unique schedule

$$\begin{gathered}p(f,v)=\frac{\Pi(v)}{-\Pi'(v)}+v, \qquad q(f,v)=\frac{-\Pi'(v)}{m(\theta(f,v))},\\ \theta(f,v)=-\frac{1}{k}\left[\Pi(v)+\Pi'(v)(f-v)\right],\end{gathered} \tag{4}$$

where the expected profit Π of seller (f,v) is characterized by the strictly convex solution to the ODE

$$\Pi'(v)=-m\left(-\frac{1}{k}\left[\Pi(v)+\Pi'(v)(\underline{f}(v)-v)\right]\right), \qquad \Pi(\underline{v})=\Pi_{CI}(\underline{f}(\underline{v}),\underline{v}). \tag{5}$$

and $\Pi_{CI}(\underline{v})$ is the complete information allocation of seller $(\underline{f},\underline{v})$.[20] To do this, I first show that the price is strictly increasing in v, and then show that the profit function Π is differentiable, which implies (4) and (5). Inspection of these equations gives properties (i), (ii), and (iii) in the theorem. To establish existence, I show that the schedule satisfies all conditions in the equilibrium definition.

Lemma 1. *Given $(f,v),(f',v')\in\tilde{S}$, in any fully separating equilibrium, $v<v'$ implies $p(f,v)<p(f',v')$. So the price function separates v.*

Proof. Suppose not. Then there exist $(f',v'),(f'',v'')\in\tilde{S}$ with $v'<v''$ such that $p(f',v')=p(f'',v'')$. Denote this common price by p. Let $A=\{(f,v)\in\tilde{S}:v\in(v',v'')\}$. Then because the price is weakly increasing in v (Lemma A.4), it must be that $p(s)=p$ for all $s\in A$. By Lemma A.3, seller profit Π depends only on v and is continuous, so it must be that $m(\theta(f,v))q(f,v)=\Pi(v)/(p-v)$ is also continuous in v and constant in f. Therefore, by Corollary A.1, $-\Pi'(v)=m(\theta(f,v))q(f,v)=\Pi(v)/(p-v)$ over A. Solving this differential equation shows that $\Pi(v)=C(p-v)$ over A for some constant C, and therefore $m(\theta(f,v))q(f,v)=C$ for all $(f,v)\in A$. So the buyer zero profit function gives $\theta(f,v)=C[f-p]/k$, and therefore $q(f,v)=C/m(\theta(f,v))=C/m(C[f-p]/k)$ for all $(f,v)\in A$.

20. See Section A.1 of the online appendix.

But then for any $(f, v_1), (f, v_2) \in A$, with $v_1 \neq v_2$, it must be that $q(f, v_1) = q(f, v_2)$ and $p(f, v_1) = p = p(f, v_2)$, contradicting full separation. $\square$

Lemma 2. *In any fully separating equilibrium, $q(\underline{f}(v), v) = 1$ for all $v \in V$.*

Proof. See the Online Appendix. $\square$

Given Lemmas 2 and buyer zero-profit, we must have $\Pi(v) = m(\theta(\underline{f}(v), v))(\underline{f}(v) - v) - k\theta(\underline{f}(v), v)$. Let $\Pi_{CI}(v) = m(\theta_{CI}(v))(\underline{f}(v) - v) - k\theta_{CI}(v)$, where $\theta_{CI}(v)$ is the unique maximizer of $m(\theta)(\underline{f}(v) - v) - k\theta$ (See Section A.1 of the online appendix). Then given $v \in V$, for any $\Pi \in (0, \Pi_{CI}(v))$, the concavity of $m(\theta)(\underline{f}(v) - v) - k\theta$ implies that there exist exactly two values of θ that solve $\Pi = m(\theta)(\underline{f}(v) - v) - k\theta$, which I denote by $\theta_U(\Pi, v)$ and $\theta_D(\Pi, v)$, where $\theta_U(\Pi, v) > \theta_D(\Pi, v)$.

Lemma 3. *For all $v > \delta \underline{f}$, $\underline{\theta}(v) \equiv \theta(\underline{f}(v), v)$ is continuous in v and equal to $\theta_D(\Pi(v), v)$.*

Proof. See the Online Appendix. $\square$

By Lemmas 2, 3, and Corollary A.1, $\Pi(\cdot)$ is differentiable for all $v > \delta \underline{f}$, so $\Pi(v) = m(\underline{\theta}(v))(\underline{f}(v) - v) - k\underline{\theta}(v) = -\Pi'(v)(\underline{f}(v) - v)) - k\underline{\theta}(v)$. Solve for $\underline{\theta}(v)$ to get $\underline{\theta}(v) = 1/k[-\Pi(v) - \Pi'(v)(\underline{f}(v) - v)]$. Applying $m(\cdot)$ to both sides and setting $-\Pi'(v) = m(\underline{\theta}(v))$ gives (5). By Lemma A.3 and the differentiability of $\Pi(\cdot)$ for all $v > \underline{v}$, we have $-\Pi'(v) = m(\theta(f, v))q(f, v)$, which gives $q(f, v) = -\Pi(v)/m(\theta(f, v))$. Also, $\Pi(v) = m(\theta(f, v))q(f, v)(p(f, v) - v) = -\Pi'(v)(p(f, v) - v)$, which implies $p(f, v) = \Pi(v)/[-\Pi'(v)] + v$. Because $\Pi(v) = m(\theta(f, v))q(f, v)(p(f, v) - v) = m(\theta(f, v))q(f, v)(f - v) - k\theta(f, v) = -\Pi'(v)(f - v) - k\theta(f, v)$, solving for $\theta(f, v)$ gives the expression in (4). Lemma A.13 in the Online Appendix establishes the boundary condition $\underline{\theta}(\underline{v}) = \theta_{CI}(\underline{v})$, which implies $\Pi(\underline{v}) = \Pi_{CI}(\underline{v})$.

Having established uniqueness, I now turn to existence, by showing that the proposed schedule satisfies the equilibrium definition conditions. The schedule satisfies Lemma A.3, and therefore satisfies seller optimality (i). By construction, it also satisfies buyer zero-profit (ii)(a). It remains to check (ii)(b), that buyers cannot profit from offering a

$(p, q) \notin M$. The set of prices posted in equilibrium is the closed interval $[\underline{p}, \bar{p}]$, where I let $\underline{p} \equiv p(\underline{v})$ and $\bar{p} \equiv p(\bar{v})$; for any $p \in [\underline{p}, \bar{p}]$, the set of quantities q posted is the closed interval $[\underline{q}(p), 1]$, where $\underline{q}(p) \equiv q(\bar{f}(v), v)$ and $p = p(v)$. Because M takes this form, it suffices to show that $p \notin [\underline{p}, \bar{p}]$ is not a profitable deviation, and that if p is an element of $[\underline{p}, \bar{p}]$, then $q < \underline{q}(p)$ is not a profitable deviation. The proof that $p \notin [\underline{p}, \bar{p}]$ is not profitable is similar to Chang (2018); for completeness, I establish it in Lemma A.14 in the online appendix. So suppose that buyers consider posting a (p, q) pair where $p \in [\underline{p}, \bar{p}]$, but $q < \underline{q}(p)$. Then by Lemma A.5 buyers expect a subset of types v such that $p(v) = p$, so then $m(\theta(p, q))q(p - v) = \Pi(v) = m(\theta(f, v))q(f, v)(p(v) - v)$, which implies $m(\theta(p, q))q = m(\theta(f, v))q(f, v)$. This gives $m(\theta(p, q)) = m(\theta(f, v))q(f, v)/q > m(\theta(f, v))$, because $q < \underline{q}(p) \leq q(f, v)$. Therefore, $\theta(p, q) > \theta(f, v)$, and because $n(\theta) = m(\theta)/\theta$ is strictly decreasing, I have $k = n(\theta(\bar{f}(v), v))q(\bar{f}(v), v)(\bar{f}(v) - p) > n(\theta(p, q))q(E[\tilde{f}|p, q] - p)$, so $(p, q) \in [\underline{p}, \bar{p}] \times [0, \underline{q}(p))$ is not a profitable deviation. □

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